Kittel TP

7.3(a). $U_0 = \frac{3}{5} N \varepsilon_F$, $\varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$.

$$p = -\left(\frac{\partial U_0}{\partial V}\right)_N = -\frac{3}{5} N \frac{\partial \varepsilon_F}{\partial V}.$$

$$\frac{\partial \varepsilon_F}{\partial V} = \frac{\hbar^2}{2m}\frac{2}{3}\left(\frac{3\pi^2 N}{V}\right)^{-1/3}\left(-\frac{3\pi^2 N}{V^2}\right)$$

$$= -\frac{\hbar^2}{3m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}\frac{1}{V}$$

$$= -\frac{2}{3}\frac{\varepsilon_F}{V}.$$

$$\Rightarrow p = -\frac{3}{5} N \frac{\partial \varepsilon_F}{\partial V} = \frac{3}{5} N \frac{2}{3}\frac{\varepsilon_F}{V}$$

$$= \frac{2}{5} N \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}\frac{1}{V}.$$

$$= \frac{N}{5m}\hbar^2\frac{1}{V}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$

$$= \boxed{\frac{\hbar^2}{5m}(3\pi^2)^{2/3}\left(\frac{N}{V}\right)^{5/3}}$$

Davidson Cheng

1.7.2024.

Kittel TP.

7.3(b) We apply thermodynamic relation

$$\tau \cancel{\#} = \left(\frac{dU}{d\sigma}\right)_{V,N} \Rightarrow \frac{1}{\tau} dU = d\sigma$$

$$\Rightarrow \quad d\sigma = \frac{1}{\tau}\left(\frac{dU}{d\tau}\right)_V d\tau$$

$$= \frac{1}{\tau} C_V \, d\tau.$$

In equation (7.34), we have $C_{el} = \frac{1}{3}\pi^2 D(\epsilon_F)\tau$ as the heat capacity of electron gas valid for $\tau \ll \tau_F$.

$$\Rightarrow \quad \sigma(\tau) = \int_0^\tau \frac{1}{\tau} C_V \, d\tau'$$

$$= \int_0^\tau \frac{1}{3}\pi^2 D(\epsilon_F) \, d\tau'$$

$$= \frac{1}{3}\pi^2 D(\epsilon_F)\tau + c.$$

Since $\sigma(\tau = 0) = 0$, the constant of integration is 0.

$\Rightarrow$. $\sigma(\tau) = \frac{1}{3}\pi^2 D(\epsilon_F)\tau$ for $\tau \ll \tau_F$.

$$= \frac{1}{3}\pi^2 \frac{3N}{2\epsilon_F}\tau$$

$$= \frac{\pi^2 N}{2\epsilon_F}\tau = \boxed{\frac{\pi^2 N}{2}\left(\frac{\tau}{\tau_F}\right).}$$

Davidson Cheng
1.7.2024.